**Calculus III for CS Project Write-ups**

1. The Hilbert Matrix
   1. Hilbert:
   2. IMPORTANT: the use of an inverse matrix function should be avoided, your program should use backward or forward substitution; use of an inverse matrix defeats the purpose of these methods (Why?).(e) Summarize your findings by plotting the errors obtained as a function of n, for each of the methods. The plots can be done using your own code, Excel or any graphing program. The plots should be included in the written component of this part of the project. (f) Answer the following questions in the associated written component for this part of the project: (i) Why is it justified to use the LU or QR-factorizations as opposed of calculating an inverse matrix? (ii) What is the benefit of using LU or QR-factorizations in this way? (Your answer should consider the benefit in terms of conditioning error.)
2. Convolution Codes
3. Urban Population Dynamics
   1. The Leslie Matrix describes the growth of populations and projected age distributions of an initial population. Each column of the matrix contains data about a certain age group of the population. The first row of each column of data in the Leslie Matrix contains data about the fecundity*, fx*, or the per capita average number of female offspring reaching the next generation born from a mother of the current generation from that age group. Below the first row, all elements in that column are zero except for one unique row. None of the other columns in the matrix will have data in that row since those columns will represent a different group. The data in this unique row represents a survival fraction for that portion of the population to go onto the next generation.
      1. For example, take the first vector (column zero) of the given Leslie Matrix. It contains the data [0, 0.7, 0, 0, 0, 0, 0, 0, 0]t. The first element contains *fx*, the fecundity of this portion of the population. The second element is nonzero, so it is the unique element whose data represents a survival fraction of the population. Given a vector v whose data represents the population at a certain date, only the first element will be multiplied by this survival fraction (0.7) and carried into the next generation. This means that 70% of the existing population of the first vector’s age class will be carried forth into the next generation. This process continues and is the same for all age classes.
      2. As this Leslie Matrix is used to model human population growth in a city, there are many factors that affect the data in its elements.
         1. Firstly, we can consider the age groups – older, higher aged humans are less likely to be alive 10 years afterwards and thus are less likely to survive to the next generation. This translates into smaller survivability fractions in the matrix for those age groups, whereas there are higher survivability fractions for younger people and middle aged people. However, more people die in their youngest years (0-10) than in their middle aged years (20-40) because of factors that include medical complications in birth, social related stress such as academics and friendships, and accidents within an individual’s control.
         2. Second, we can consider the fecundity of each age group. Conceivably, there are no 0-10 year olds who are capable of producing children so *fx* is 0 for them. In addition, as fertility decreases with age past the middle years, *fx* declines from 0.9 for 30-40 age class to 0.1 for the 40-50 age class to 0 for the 50-60 age class. The older age classes have little to no chance of producing offspring at their ages to pass on to the next generation, while middle aged groups have a chance of producing 1 or even more than 1 children. These are the years that children become adults and lead more financially and socially independent lives, often marrying and obtaining jobs, and humans are most fertile at this age, so it makes sense that most people would be having children at this age.
   2. The population distributions for 2010, 2020, 2030, 2040, and 2050 are as follows:
      1. Initial: [ 210000, 190000, 180000, 210000, 200000, 170000, 120000, 90000, 50000]^t
      2. 2010: [ 635000, 147000, 161500, 162000, 189000, 176000, 136000, 92400, 36000]^t
      3. 2020: [ 518750, 444500, 124950, 145350, 145800, 166320, 140800, 104720, 36960]^t
      4. 2030: [ 816240, 363125, 377825, 112455, 130815, 128304, 133056, 108416, 41888]^t
      5. 2040: [ 965648.5, 571368, 308656.25, 340042.5, 101209.5, 115117.2, 102643.2, 102453.12, 43366.4]^t
      6. 2050: [1341322.675, 675953.95, 485662.8, 277790.625, 306038.25, 89064.36, 92093.76, 79035.264, 40981.248]^t