**Calculus III for CS Project Write-ups**

1. The Hilbert Matrix
2. IMPORTANT: the use of an inverse matrix function should be avoided, your program should use backward or forward substitution; use of an inverse matrix defeats the purpose of these methods (Why?).

e. The chart below summarizes the errors for LU, Householder and Givens factorizations. The errors have been plotted as a function of *n*, e.g. a 2.0 on the x-axis corresponds to the 2x2 Hilbert matrix. All the errors are very small. Nonetheless, LU factorization yielded a consistent error 0 for every Hilbert matrix (2x2 – 20x20).

INSERT PLOTS HERE.

1. i) By using LU or QR-factorizations, we can solve the problem with forward and backward substitution. This process involves several numbers of simple algebraic operations directly proportional to the size of the given matrix. On the other hand, calculating the inverse matrix would not be easily formulated into a procedure that can give to a computer. If one did create such a procedure, it would require many more multiplication and division operations, which would increase the final error due to the finite precision of Double rounding errors.

ii) Using LU or QR-factorizations does not incur significant error, as seen in the graphs above. As n increases, there is little change in the error of (LU – H)/(QR – H) and (Hx – b), meaning that these factorizations can be used effectively when scaled to higher n values. The increase in error, which was recorded to be no larger than 1E-12 for (Hx - b), is worth it due to a decrease in runtime. The conditioning error is minimized as much as possible. Conditioning is a property of the matrix A that determines whether it is feasible for an algorithm to provide a numerical solution to a linear system involving A. Since any linear system on a computer has some error, it is important to reduce perturbation or imprecision as much as possible. LU and QR help to accomplish this. Unfortunately, LU is prone to instability, meaning that if both *L* and *U* do not have small condition numbers, then instability in the calculated answer will be inevitable.

1. Convolution Codes

For the written component of this part, compare the results of the two methods above, and discuss the number of iterations required to obtain the desired precision. Is the length of the initial stream n important? Does n have an effect on the number of iterations required to achieve the error tolerance?

1. Urban Population Dynamics
   1. The Leslie Matrix describes the growth of populations and projected age distributions of an initial population. Each column of the matrix contains data about a certain age group of the population. The first row of each column of data in the Leslie Matrix contains data about the fecundity*, fx*, or the per capita average number of female offspring reaching the next generation born from a mother of the current generation from that age group. Below the first row, all elements in that column are zero except for one unique row. None of the other columns in the matrix will have data in that row since those columns will represent a different group. The data in this unique row represents a survival fraction for that portion of the population to go onto the next generation.
      1. For example, take the first vector (column zero) of the given Leslie Matrix. It contains the data [0, 0.7, 0, 0, 0, 0, 0, 0, 0]t. The first element contains *fx*, the fecundity of this portion of the population. The second element is nonzero, so it is the unique element whose data represents a survival fraction of the population. Given a vector v whose data represents the population at a certain date, only the first element will be multiplied by this survival fraction (0.7) and carried into the next generation. This means that 70% of the existing population of the first vector’s age class will be carried forth into the next generation. This process continues and is the same for all age classes.
      2. As this Leslie Matrix is used to model human population growth in a city, there are many factors that affect the data in its elements.
         1. Firstly, we can consider the age groups – older, higher aged humans are less likely to be alive 10 years afterwards and thus are less likely to survive to the next generation. This translates into smaller survivability fractions in the matrix for those age groups, whereas there are higher survivability fractions for younger people and middle aged people. However, more people die in their youngest years (0-10) than in their middle aged years (20-40) because of factors that include medical complications in birth, social related stress such as academics and friendships, and accidents within an individual’s control.
         2. Second, we can consider the fecundity of each age group. Conceivably, there are no 0-10 year olds who are capable of producing children so *fx* is 0 for them. In addition, as fertility decreases with age past the middle years, *fx* declines from 0.9 for 30-40 age class to 0.1 for the 40-50 age class to 0 for the 50-60 age class. The older age classes have little to no chance of producing offspring at their ages to pass on to the next generation, while middle aged groups have a chance of producing 1 or even more than 1 children. These are the years that children become adults and lead more financially and socially independent lives, often marrying and obtaining jobs, and humans are most fertile at this age, so it makes sense that most people would be having children at this age.
   2. The population distributions for 2010, 2020, 2030, 2040, and 2050 are as follows:
      1. Initial: [ 210000, 190000, 180000, 210000, 200000, 170000, 120000, 90000, 50000]^t
      2. 2010: [ 635000, 147000, 161500, 162000, 189000, 176000, 136000, 92400, 36000]^t
      3. 2020: [ 518750, 444500, 124950, 145350, 145800, 166320, 140800, 104720, 36960]^t
      4. 2030: [ 816240, 363125, 377825, 112455, 130815, 128304, 133056, 108416, 41888]^t
      5. 2040: [ 965648.5, 571368, 308656.25, 340042.5, 101209.5, 115117.2, 102643.2, 102453.12, 43366.4]^t
      6. 2050: [1341322.675, 675953.95, 485662.8, 277790.625, 306038.25, 89064.36, 92093.76, 79035.264, 40981.248]^t
   3. The largest eigenvalue of the matrix is .99999999. This means that the population will become stable in the long run, after a period of growth. It appears to converge exactly to one, which would fit the predicted growth pattern. The right eigenvectors comprise the stable age distribution and the left eigenvectors comprise the reproductive value. Since the population growth rate is positive, the population will continue to grow and eventually become stable since the age groups are stable and the reproductive values are good for the younger age groups.

d. A decrease in the birth rate of the second age group made very little impact on the eigenvalue of the function. The calculated eigenvalue ended up being 1.00000002, which is still very close to one. This means that the population remained very stable even after these changes. Since the population growth rate is positive, the population will continue to grow and eventually become stable since the age groups are stable and the reproductive values are good for the younger age groups.